

Quark matter equation of state and stellar properties

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Abstract – In this paper we study strange matter by investigating the stability window within the QMDD model at zero temperature and check that it can explain the very massive pulsar recently detected. We compare our results with the ones obtained within the MIT bag model and see that the QMDD model can explain larger masses, due to the stiffening of the equation of state.

Introduction. – Neutron star is a very dense object believed to be the remnant of a supernova explosion. Its estimated mass lies around $(1 - 2M_{\odot})$, its radius is of the order 10 km and its temperature of the order of $10^{11} K$ at birth, cooling rapidly to about $10^{10} K$ by emitting neutrinos. In most physical models for neutron stars, the star is composed only of hadrons and leptons, whose stellar matter is a mixture of degenerate neutrons, protons and electrons. In all stellar models, the structure matter depends on the assumed equation of state (EOS), which is derived from effective models. One of the uncertainties related to neutron star properties is the ground state of nuclear matter. In most models, hadrons are assumed to be the true ground state of the strong interaction. Since the proposal by Witten [1] that strange matter (SM) may be the actual ground state of baryon matter at high densities, many investigations have been conducted to verify the veracity of this hypothesis and the implications in several areas of physics, astrophysics and cosmology with possible consequences in the QCD phase diagram.

SM was first considered in calculations obtained within the MIT bag model framework [2]. More sophisticated treatments for SM, based on the Nambu-Jona-Lasinio [3] and the color flavor locked phase models [3, 4] also exist in the literature.

In [5], a confinement mechanism was introduced by assuming that the quark masses are density dependent. This model, named QMDD, was then applied to describe SM and the related quark star properties in [6]. In the very same paper, the authors pointed out that the results obtained with the QMDD model were quite different from the ones obtained with the MIT model. Subsequently, in

[7], the authors claimed that this difference was due to an incorrect thermodynamical treatment of the problem and recalculated the equation of state showing that an extra term is present in the energy density and pressure of the system. This extra term results from the dependence of the quark masses on the baryonic density.

An important ingredient in the SM hypothesis is the stability window, identified with the constant values of the model that are consistent with the fact that two-flavor quark matter must be unstable (i.e., its energy per baryon has to be larger than 930 MeV, which is the iron binding energy) and SM (three-flavor quark matter) must be stable, i.e., its energy per baryon must be lower than 930 MeV. It was also shown in [7] that the zero pressure density does not correspond to the minimum of the energy per baryon, as is normally the case, because of the quark mass density dependence. In [8], the author has shown that the pressure at the density corresponding to the minimum of the energy density can be zero if it is calculated in a self-consistent way along the energy density. That calculation requires another extra term in the grand canonical thermodynamical potential, which mimics the MIT bag constant, but instead of being constant, it increases with the increase of the density.

Still another calculation of the equation of state based on thermodynamics and also on the general ensemble theory was obtained in [9]. The authors claim that the extra term reported in [7] is correct in the expression of the pressure, but should not be present in the energy density equation. Similar kinds of density dependence are found in hadronic models [10, 11] and used to describe protoneutron star properties in [12, 13]. Specifically, in [11, 13], the

hadronic masses are density dependent while in [10,12] the coupling constants of the model are density dependent. In both approaches, the pressure also carries a rearrangement term that appears due to the density dependence, while such a term cancels out in the energy density expressions. In [9], due to quark confinement and asymptotic freedom, another prescription for the quark masses is used and the values for the quark current masses are somewhat different from the standard ones. Normally $m_{0u} = m_{0d} = 5$ MeV and m_{0s} is of the order of 150 MeV. In [9], $m_{0u} = 5$ MeV, $m_{0d} = 10$ MeV and $m_{0s} = 80$ or 90 MeV. Depending on the parametrization used, SM is completely stable or becomes metastable.

More recently, the QMDD model was again revisited [14]. As already explained in this Introduction, the grand canonical thermodynamical potential is not only a function of T, V and the chemical potential μ , but also depends explicitly on the quark (or related baryon) density through its mass and this fact caused all the different treatments described below in the previous papers. As pointed out in [14], the extra terms for the different treatments contradict each other and tackling thermodynamics self-consistently in the QMDD model seems to be a real problem. Another recipe is then proposed, considering an ideal quasi-particle system, where the mass can be both density and temperature dependent, but is taken as constant at fixed values of T_0 and ρ_0 . In this case, the standard ideal gas expressions are recovered and the extra terms do not appear. Moreover, the pressure of the system is always positive and this poses a problem related to the stability of SM. In order to circumvent this difficulty, the authors have to consider the physical vacuum.

In 2010, a measurement of the Shapiro delay in the radio pulsar PSR J1614-2230 yielded a mass of $1.97 \pm 0.04 M_\odot$, the highest mass of a compact object ever measured. It is well known that the stellar mass that can be supported against gravitational collapse depends on the equation of state used to describe it. According to [16], very massive stars, with masses $2M_\odot \leq M \leq 2.73M_\odot$ can be interpreted as compact stars composed entirely of deconfined quarks and would be quark stars.

The aim of the present work is to check whether this high mass pulsar can be described by the QMDD model. A necessary investigation that precedes the use of the equation of state as input to the TOV equations [17] is the detailed consideration of the stability window. In the present paper we consider quarks with equal chemical potentials, closer to the situation in ^{56}Fe where matter is almost symmetric, than to the situation of charge neutrality usually analysed, where electrons, not present in nuclei, are included. Whenever possible, our results are compared with the MIT model. In face of all the discussions on the thermodynamical consistency of different versions of the QMDD model, we restrict ourselves to the version obtained in [7].

Quark matter. – In this section we summarize the models we use to describe the properties of quark matter. The QMDD model is based on a phenomenological approach [7] where the dynamical masses of the three lightest quarks scale inversely with the baryon number density [6], what mimics an interaction among the quarks:

$$m_{u,\bar{u}}^* = m_{d,\bar{d}}^* = \frac{C}{3n_B}, \quad m_{s,\bar{s}}^* = m_{0s,\bar{s}} + \frac{C}{3n_B}, \quad (1)$$

where C is an energy density constant in the zero quark density limit,

$$n_B = \frac{1}{3} (\rho_u + \rho_d + \rho_s), \quad (2)$$

is the baryon number density and $m_{0s,\bar{s}}$ is the current mass of the s quark. The energy density, the pressure and the quark density are respectively given by

$$p = \sum_q \frac{\gamma_q}{2\pi^2} \int_0^\infty \frac{p^2}{E_q^*(p)} \left[\frac{p^2}{3} - \frac{C}{3n_B} \right] (f_{q+}^* + f_{q-}^*) dp, \quad (3)$$

$$\varepsilon = \sum_q \frac{\gamma_q}{2\pi^2} \int_0^\infty \frac{p^2}{E_q^*(p)} \left[E_q^{*2}(p) + \frac{C}{3n_B} \right] (f_{q+}^* + f_{q-}^*) dp, \quad (4)$$

$$\rho_q = \frac{\gamma_q}{2\pi^2} \int_0^\infty p^2 (f_{q+}^* - f_{q-}^*) dp, \quad (5)$$

where $q = u, d, s$, and m_q^* is the effective quark mass. The distribution functions for quarks and anti-quarks are the Fermi-Dirac distributions $f_{q\mp} = 1 / (1 + \exp[(E_q^*(p) \mp \mu_q) / T])$, with the chemical potential for quarks (upper sign) and anti-quarks (lower sign) and $E_q^*(p) = \sqrt{p^2 + m_q^{*2}}$. For $T = 0$, there are no antiparticles, the chemical potential is equal to the Fermi energy, and the distribution function for the particles is the usual step function: $f_{q\mp} = \Theta_q(p_f^2 - p^2)$.

In the description of compact stars, both charge neutrality and β equilibrium conditions have to be imposed [3]:

$$2\rho_u = \rho_d + \rho_s + 3(\rho_e + \rho_\mu) \quad (6)$$

and

$$\mu_s = \mu_d = \mu_u + \mu_e, \quad \mu_e = \mu_\mu. \quad (7)$$

For the electron and muon pressure, energy density and densities we just replace $q \rightarrow l$ in equations (3),(4) and (5), where $l = e, \mu$ and $\gamma_l = 2$.

We next summarize the main formulae at $T = 0$ for both models used in this paper, the QMDD model [7] and MIT bag model [2].

The energy density, the pressure, the quark density and the baryonic density given in eqs. (3),(4) and (5) can be rewritten respectively as:

$$p = \sum_q \frac{\gamma_q m_q^{*4}}{48\pi^2} F(x_q^*) - B(C, m_q^*), \quad (8)$$

$$\varepsilon = \sum_q \frac{\gamma_q m_q^{*4}}{48\pi^2} 3H(x_q^*) + B(C, m_q^*), \quad (9)$$

$$\rho_q = \frac{\gamma_q}{6\pi^2} m_q^{*3} x_q^3, \quad (10)$$

$$n_B = \frac{1}{3} \sum_q \frac{\gamma_q}{6\pi^2} m_q^{*3} x_q^3, \quad (11)$$

where the dynamic term of confinement is given by:

$$B(C, m_q^*) = \sum_q \frac{\gamma_q m_q^{*4}}{48\pi^2} \frac{C}{n_B} \left(\frac{4}{m_q^*} \right) G(x_q^*), \quad (12)$$

and the functions

$$F(x_q^*) = x_q^* (x_q^{*2} + 1)^{1/2} (2x_q^{*2} - 3) + 3 \ln \left[(x_q^{*2} + 1)^{1/2} + x_q^* \right], \quad (13)$$

$$H(x_q^*) = x_q^* (x_q^{*2} + 1)^{1/2} (2x_q^{*2} + 1) - \ln \left[(x_q^{*2} + 1)^{1/2} + x_q^* \right], \quad (14)$$

$$G(x_q^*) = x_q^* (x_q^{*2} + 1)^{1/2} - \ln \left[(x_q^{*2} + 1)^{1/2} + x_q^* \right], \quad (15)$$

and

$$x_q^* = \left[\left(\frac{\mu_q}{m_q^*} \right)^2 - 1 \right]^{1/2}. \quad (16)$$

Note that the term $B(C, m_q^*)$ arises from the derivatives of the grand canonical thermodynamical potential of a Fermi gas with respect to m^* [16]. This density dependent term leads to a thermodynamical inconsistency, because the fundamental relation of thermodynamics $\Omega = -pV$ is violated. Although the model has this problem, we note that this density dependent term leads to an empirical model incorporation of the quark confinement at the low density regime and to asymptotic freedom at very high densities, as already emphasized in the Introduction.

For the usual MIT bag model [2] the quark masses are fixed as $m_u = m_d = 5$ MeV, $m_s = 150$ MeV and its expressions are (8),(9), (10) where m^* is simply replaced by m and $B(C, m_q^*)$ by a bag constant B .

Results and conclusions. — We now want to establish the conditions under which SM is the true ground state [1, 7]. In principle, the theory of the strong interaction should contain the answer to the question of whether strange matter is stable. Unfortunately, as we all know, QCD is still far from being completely solved. We need to rely on effective models to test the idea of SM. Hence, we next study strange matter via two different effective models and find a sizeable region of parameter space for which strange matter is stable. For the QMDD case, these parameters are C and the current strange quark mass m_{0s} (see equation (1)). For the MIT bag model, these parameters are the well-known $B^{1/4}$ and current strange quark mass m_{0s} . Strange matter is stable at zero pressure if its energy density per baryon density is lower than the energy per baryon of ^{56}Fe , $\left(\frac{\varepsilon}{n_B} = \frac{E}{A} \right)_{SM} \leq 930 \text{ MeV}$ while the two-flavor quark matter must be unstable (i.e., its energy per baryon has to be larger than 930 MeV). Notice that, if finite size effects are taken into account, this value is around 4 MeV lower, i.e. 934 MeV [2], but the final stability window remains practically unchanged. The numbers are discussed below. The energy per nucleon has been calculated numerically for 2QM and SM respectively. Therefore, SM is stable in the shaded region shown in Fig. 1, for the QMDD model and in Fig. 2 for the MIT model. The lower limit, vertical straight line, is due to the requirement that two flavor quark matter is not absolutely stable. In both figures one can see that we have allowed the strange quark mass to vary considerably. We have checked that the results are slightly different if we consider matter with identical quark chemical potentials, corresponding to symmetric matter in the 2QM or charge neutral matter in β -equilibrium, as expected in stars. For both models, the stable region is larger if the conditions given in eqs.(6) and (7) are imposed. This is the situation which is usually analysed in the literature, but one has to bear in mind that stable nuclear matter (as in iron) does not obey these conditions and no electrons are present. Actually, its proton fraction is $Y_p = 0.46$, very close to symmetric matter and this is the reason why we have chosen to analyse matter with equal quark chemical potentials. If we consider finite size effects and take $m_{0s} = 150$ MeV, the border of the stability line moves from $C = 80.1$ to 79.2 MeV/fm³ in the QMDD model and from $B^{1/4} = 155.1$ to 155.8 in the MIT model.

In Fig. 3a, we plot the equations of state (EoS) obtained for two constants in the MIT model and one situation in the QMDD model, chosen from the values which satisfy the SM stability condition. For these choices of parameters, the QMDD EoS is reasonably stiffer. The MIT bag model presents a linear dependence of the pressure against the energy density as shown in Fig. 3. The parameter B is just a linear parameter in the EOS for the MIT case. This is not true for the QMDD model, because the pressure as a function of the energy density is more complicated, as seen in Fig. 3. This non-trivial behav-

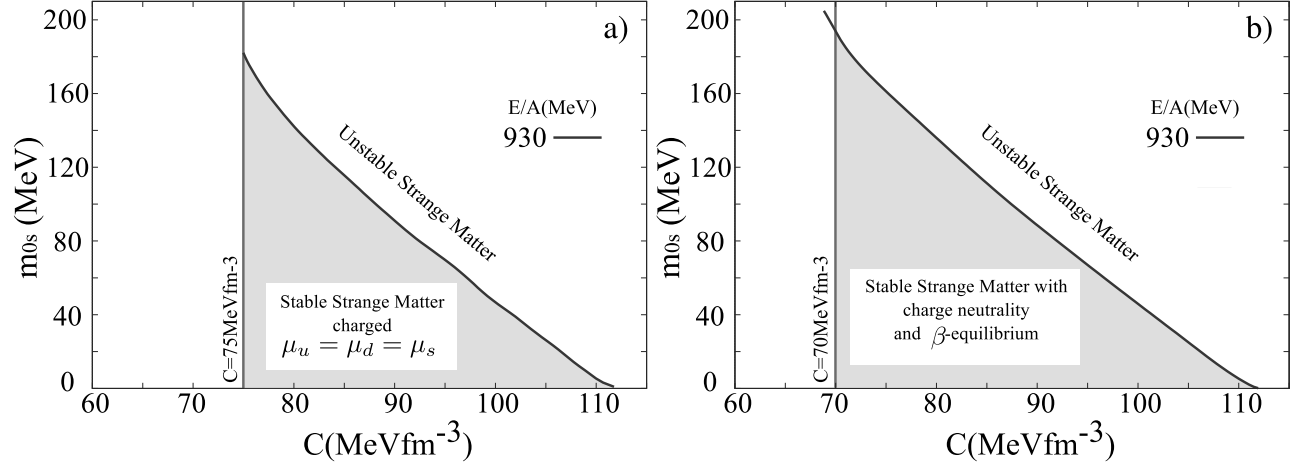


Fig. 1: SM stability window obtained with the QMDD model at $T = 0$. The points in the figures show the maximum values for C when $m_{0s} = 150$ MeV is fixed.

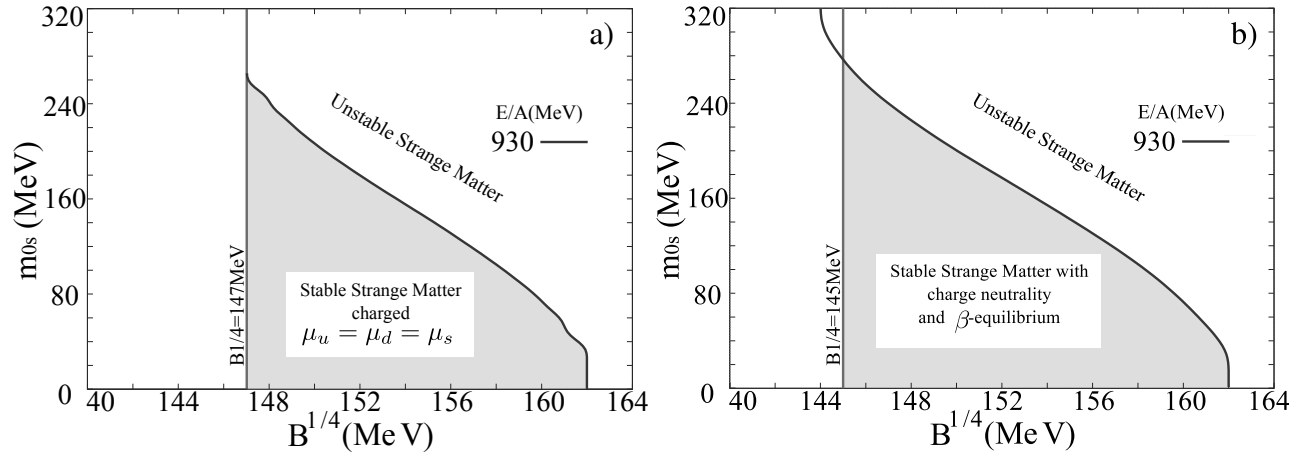


Fig. 2: SM stability window obtained with the MIT model at $T = 0$. The points in the figures show the maximum values for $B^{1/4}$ when $m_{0s} = 150$ MeV is fixed.

ior is due to $B(C, m_q^*)$ in equation (12). If we consider the thermodynamic treatment used in [9], where the zero pressure point corresponds to the minimum of the energy per baryon, the stability window modes to larger parameters, as seen in Fig. 3b and the equations of state become softer.

As stated in the Introduction, in order to study hydrostatic equilibrium of a family of quark stars we have to solve numerically the Tholman-Oppenheimer-Volkoff equations (TOV) [17]. The mass radius relation is obtained from the solution of these coupled differential equations.

The determination of small radii as the ones expected in pulsars from observations based on luminosity and temperature is full of uncertainties [18]. Hence, although X-ray astronomy has provided some simultaneous determination of masses and radii from X-ray bursters [19], they have to be taken with care. The integration of the TOV equations gives quark stars whose mass-radius relation is shown in Fig. 4. In this figure the horizontal lines are the masses of some well-known pulsars extracted from [15,20]. The lower and upper limits of the masses and radii of EXO 0748-676 and 4U 1608-52 are also displayed. The shaded clouds refer to the 1σ and 2σ confidence ellipse of the results obtained in [19] for the EXO 1745-248. We notice that the QMDD model can certainly describe compact objects which are more massive than the MIT model, as already expected from its harder EoS. Nevertheless, it fails to describe pulsars with low radii, which can be described by the MIT model. One, however, has to bear in mind the uncertainties discussed in [18]. In Fig. 5, we show the maximum masses obtained in different points of the stability window for both models. They increase with the decrease of the strange quark mass and change vary little for the parameters on the right border line. For the present EOS, the QMDD model yields systematically larger maximum masses than the MIT model.

All the equations of state obtained for the QMDD model discussed in the Introduction rely on derivatives of the grandcanonical thermodynamical potential, which are truncated at different arbitrary points. To obtain the correct expressions, an alternative calculation can be done via the energy-momentum tensor as [10,12,13]. This work is in progress.

In a forthcoming paper, we will analyse carefully the stability window for protoquark stars described by quark matter at finite temperature (or fixed entropy). The choice of appropriate parameters in the search for stable SM at finite temperature systems requires, in addition to the investigation presented in this paper, a careful study of the free energy per baryon ($\mathcal{F} = \varepsilon - TS$, where S is the entropy density of the system), in contrast to previous works [21–24], where the energy density per baryon was used. Of course, the lower limits, given by the straight lines on the left of figures 1 and 2 remain the same because the ^{56}Fe ground state is a zero temperature system, where the binding energy and the free energy per baryon coincide.

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REFERENCES

- [1] WITTEN E., *Phys. Rev. D*, **30** (1984) 272.
- [2] CHODOS A., JAFFE R.L., JOHNSON K., THORNE C.B. AND WEISSKOPF V.F., *Phys. Rev. D*, **9** (1974) 3471; FARHI E. AND JAFFE R.L., *Phys. Rev. D*, **30** (1984) 23792390.
- [3] MENEZES D.P., PROVIDÊNCIA C. AND MELROSE D.B., *J. Phys. G: Nucl. Part. Phys.*, **2006** (1081-1095) ; MENEZES D.P. AND MELROSE D.B., *Publ. Astr. Soc. Aust.*, **22** (2005) 292-297.
- [4] PAULUCCI L., FERRER E.J., DE LA INCERA V. AND HORVATH J.E., *Phys. Rev. D*, **83** (2011) 043009; OLIVEIRA J. C., RODRIGUES H. AND DUARTE S. B., *Phys. Rev. D*, **78** (2008) 123008; ORSARIA M., RODRIGUES H. AND DUARTE S.B., *Int. J. Mod. Phys. D*, **16** (2007) 175.
- [5] FOWLER G.N., RAHA S. AND WEINER R.M., *Z. Phys. C*, **9** (1981) 271-273.
- [6] CHAKRABARTY S., *Phys. Rev. D*, **43** (1991) 627.
- [7] BENVENUTO O.G. AND LUGONES G., *Phys. Rev. D*, **51** (1995) 1989.
- [8] WANG P., *Phys. Rev. C*, **62** (2000) 015204.
- [9] PENG G.X., CHIANG H.C., ZOU B.S., NING P.Z. AND LUO S.J., *Phys. Rev. C*, **62** (2000) 025801.
- [10] TYPEL S. AND WOLTER H.H., *Nucl. Phys. A*, **656** (1999) 331.
- [11] BROWN G. E. AND RHO M., *Phys. Rev. Lett*, **66** (1991) 2720.
- [12] AVANCINI S.S., BRACCO M.E., CHIAPPARINI M. AND MENEZES D.P., *Phys. Rev. C*, **67** (2003) 024301.
- [13] AVANCINI S.S. AND MENEZES D.P., *Phys. Rev. C*, **74** (2006) 015201.
- [14] YIN S. AND SU R.K., *Phys. Rev. C*, **77** (2008) 055204.
- [15] DEMOREST P. B., PENNUCCI T., RANSOM S. M., ROBERTS M. S. E. & HESSELS J. W. T., *Nature*, **467** (2010) 10811083.
- [16] OLIVEIRA J. C., RODRIGUES H., AND DUARTE S. B., *ApJ*, **730** (2011) 31.
- [17] R.C. TOLMAN, *Phys. Rev*, **55** (1939) 364; OPPENHEIMER J.R. AND VOLKOFF G.M., *Phys. Rev*, **55** (1939) 374.
- [18] SULEIMANOV V., POTEKHIN A. Y., AND WERNER K., *Adv. Space Res.*, **45** (2010) 92-98.
- [19] ÖZEL F., BAYM G., AND GUVER T., *Phys. Rev. D*, **82** (2010) 101301; ÖZEL F., GÜVER T., PSALTIS D., *ApJ*, **693** (2009) 1775; ÖZEL F., *Nature*, **441** (2006) 1115.
- [20] ZHANG C. M., WANG J., ZHAO Y. H., YIN H. X., SONG L. M., MENEZES D. P., WICKRAMASINGHE D. T., FERRARIO L., AND CHARDONNET P., *A&A*, **527** (2011) 527-534.
- [21] CHMAJ T. AND SLOMINSKI W., *Phys. Rev. D*, **40** (1988) 165.
- [22] CHAKRABARTY S., *Phys. Rev. D*, **48** (1993) 1409.
- [23] LUGONES G. AND BENVENUTO O.G., *Phys. Rev. D*, **52** (1995) 1276.
- [24] ZHANG Y. AND SU R.K., *Phys. Rev. C*, **65** (2002) 035202

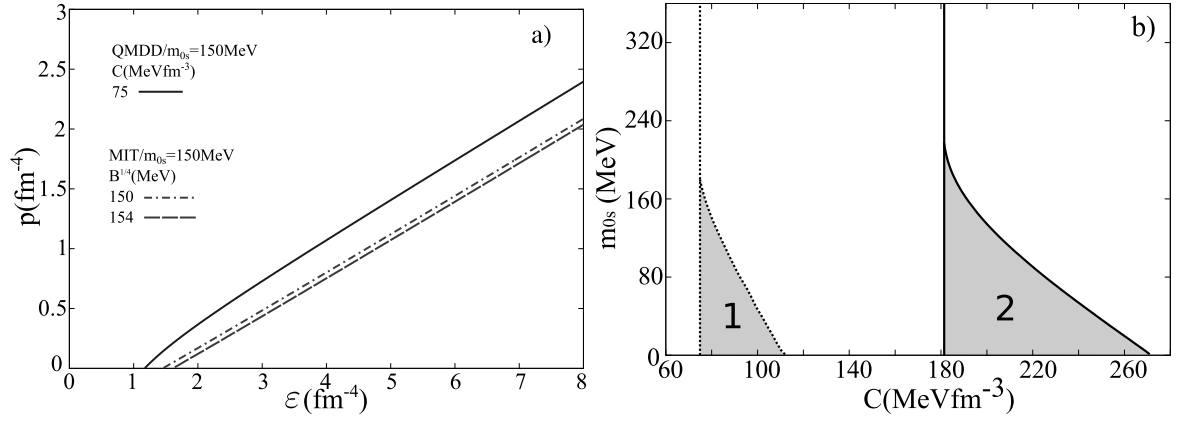


Fig. 3: In a) we show strange matter equations of state for 3 parameter choices and in b) we compare the stability window for two different versions of the QMDD model.

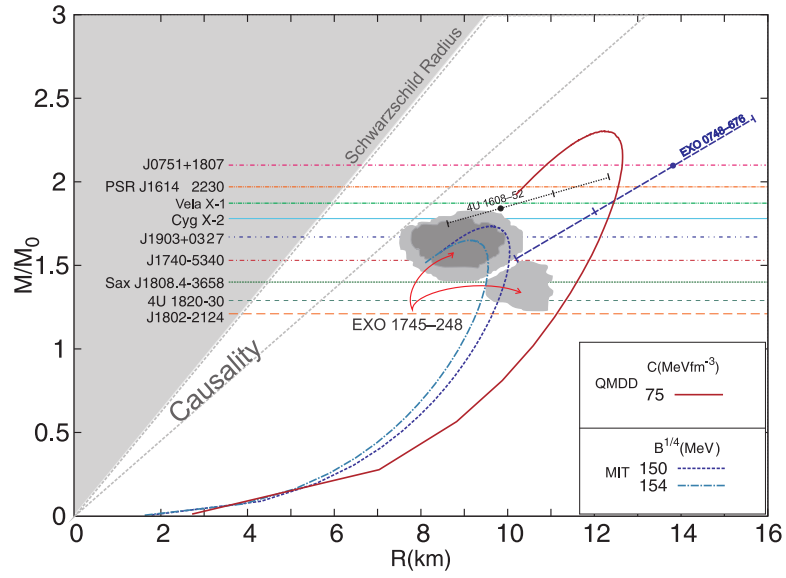


Fig. 4: Mass-radius relations for different strange stars.

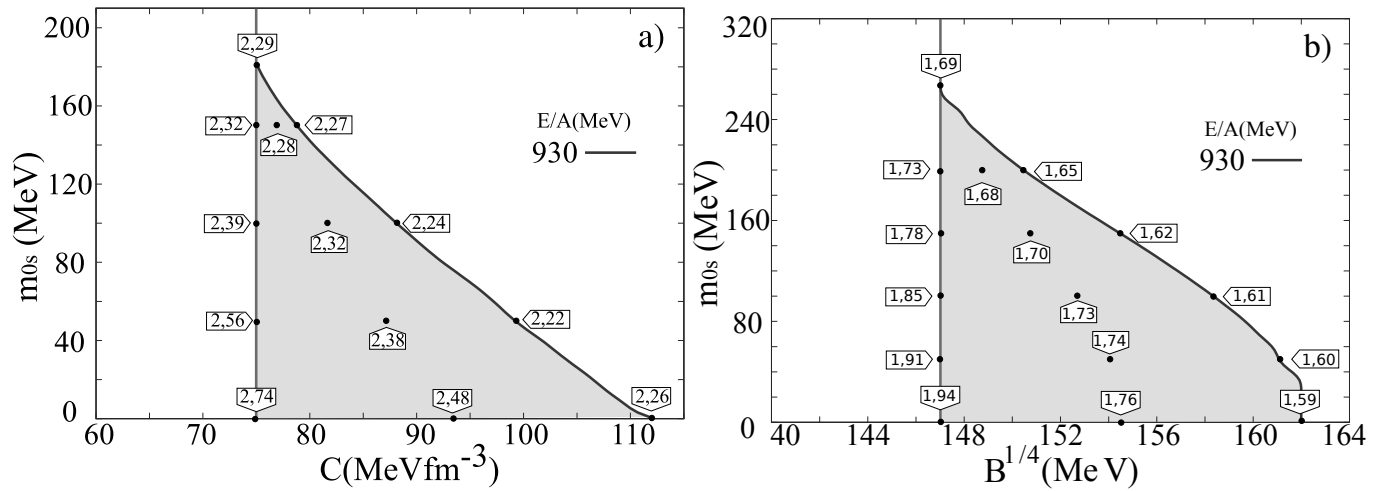


Fig. 5: Inside the flags we show the maximum masses (M/M_\odot) for the corresponding parameter sets inside the stability window for a) the QMDD and b) the MIT models respectively.